

Comment on “A counterpart of the WKI soliton hierarchy associated with $\mathfrak{so}(3, \mathbb{R})$ ”

Takayuki TSUCHIDA

June 13, 2014

In the recent paper (Wen-Xiu Ma, Solomon Manukure and Hong-Chan Zheng, arXiv:1405.1089), the authors proposed an integrable hierarchy different from the well-known Wadati–Konno–Ichikawa (WKI) hierarchy. However, using a simple linear change of dependent variables such as

$$p + iq =: u, \quad p - iq =: v,$$

one can check that their hierarchy is equivalent to the WKI hierarchy. Note, for example, that the second nontrivial flow in their hierarchy ((3.18) in arXiv:1405.1089),

$$\begin{bmatrix} p \\ q \end{bmatrix}_{t_1} = - \begin{bmatrix} \left(\frac{p_x}{(p^2+q^2+1)^{\frac{3}{2}}} \right)_{xx} \\ \left(\frac{q_x}{(p^2+q^2+1)^{\frac{3}{2}}} \right)_{xx} \end{bmatrix},$$

is transformed to the corresponding flow in the WKI hierarchy:

$$\begin{bmatrix} u \\ v \end{bmatrix}_{t_1} = - \begin{bmatrix} \left(\frac{u_x}{(1+uv)^{\frac{3}{2}}} \right)_{xx} \\ \left(\frac{v_x}{(1+uv)^{\frac{3}{2}}} \right)_{xx} \end{bmatrix}.$$

For the same reason, some “new integrable” hierarchies proposed by Wen-Xiu Ma and coworkers in recent e-prints are equivalent to the already known ones.